

THE GAS PRODUCTION OF COMETS

W. F. Huebner

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16. Abstract  The gas production rates of comets, based on Whipple's icy-conglomerate model, are reinvestigated in the light of energy balance between the Sun's radiation and reradiation plus evaporation by the comet. Extinction of the Sun's radiation by dust in the cometary atmosphere, in the case of comets rich in dust, can considerably influence evaporation. The estimated gas productivities for comets in which the forbidden [OI] lines have been observed are in good agreement with the production rates calculated here. A variation of Levin's formula which relates the heat of evaporation (desorption) to the brightness of a comet, is discussed.			
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## THE GAS PRODUCTION OF COMETS

W. F. Huebner<sup>1,2</sup>

## I. INTRODUCTION

The estimates of Biermann and Trefetz (1964) indicate that gas production in comets in which the forbidden [OI]-lines are observed is much greater than was previously assumed. It is therefore interesting to check the possible production rates. It is shown that solar radiation provides sufficient energy to reach the necessary high level of evaporation. Together with the additional energy which may be liberated by chemical reactions in the vicinity of the nucleus surface, the estimated yield may even be considerably exceeded. A mutual relationship exists between the observed intensity of the forbidden [OI]-lines and the derived density of the cometary atmosphere. In the comets examined here, the red doublet is stronger than the green line if the density is high. This behavior indicates that a depopulation of the  $^1D_2$  level, from which the red doublet originates, is not likely. /22\*

A comparison of the observations with the calculated production rates also gives the heats of evaporation of the ice conglomerate. These heats of evaporation differ considerably for different comets. Some comets that have been observed for the first time consist of very volatile substances, particularly on their approach to the sun. Periodic comets, on the other hand, consist primarily of  $H_2O$ . /23

## II. ENERGY EQUILIBRIUM

All of the energy striking the comet at a distance of one astronomical unit from the Sun is equal to the solar constant  $S = 3.16 \cdot 10^{-2} \text{ cal cm}^{-2} \text{ sec}^{-1}$ .

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\*Numbers in the margin indicate pagination in the foreign text.

The energy of the solar wind in the undisturbed state, on the other hand, may be disregarded. It is assumed that the cometary nucleus rotates with a period of approximately one day, so that the solar radiation striking a cross-section  $\pi R^2$  is distributed over a time average over the entire surface  $4\pi R^2$  ( $R$  = "nuclear radius"). The average incident energy per unit time and unit surface is therefore

$$J_e = S/4 = 7.90 \times 10^{-3} \text{ cal cm}^{-2} \text{ sec}^{-1}. \quad (1)$$

The possible extinction in the cometary atmosphere can now be estimated. The ionization energies for the elements occurring in the comet can be in the vicinity of 12 eV ( $\lambda \approx 1,000 \text{ \AA}$ ) and the dissociation energies for the molecules are at 4 eV ( $\lambda \approx 3,000 \text{ \AA}$ ). In the visual wave length range, the absorption of the lines is very small due to their narrow width; the molecular bands lie primarily in the infrared. The radiation absorption cross-section for ionization, dissociation and the lines (blurred for several  $10^3 \text{ \AA}$ ) is of the order of magnitude of  $\sigma_a \sim 10^{-17} \text{ cm}^2$  and the optical depths is therefore

$$\int_R^\infty \mu dr = \int_R^\infty \sigma_a \frac{Q}{v r^2} dr = \frac{\sigma_a Q}{v R} \approx 10^2. \quad (2)$$

with  $Q/v \approx 10^{25}$  molecules per spatial angle and cm along the radius and  $R \approx 10^6 \text{ cm}$ . The radial outflow velocity of the molecules in the cometary atmosphere is  $v \approx 10^5 \text{ cm sec}^{-1}$  and  $Q$  is the molecular yield of the comet per spatial angle and unit time. In view of the great optical depth, it may be assumed that all radiation with  $\lambda < 2,000 \text{ \AA}$  is completely absorbed in the cometary atmosphere. /24

Free-free absorption has the cross section  $\sigma_{ff} < 10^{-30} \text{ cm}^2$ , and may therefore be disregarded. Thomson scattering by free electrons has a cross section of  $0.66 \cdot 10^{-24} \text{ cm}^2$  and if we assume an electron density of  $10^6 \text{ cm}^{-3}$  to be constant over the entire coma up to a radius of  $R = 10^{10} \text{ cm}$ , the corresponding optical depth would always be less than  $10^{-8}$ .

Rayleigh scattering by molecules is only important for wave lengths  $\lambda$  which are much greater than the average distance between the molecules, so that for

$$\lambda \gg l \approx n_0^{-1/3} = (cR^2/\rho)^{1/3} \approx 5 \times 10^{-5} \text{ cm}, \quad (3)$$

where  $n_0$  is the molecular density on the surface of the nucleus.

In a dust free comet, therefore, we have complete extinction for  $\lambda < 2,000 \text{ \AA}$ , some absorption and scatter in the infrared, and naturally diffuse reflection on the surface of the nucleus (Albedo). It may therefore be assumed that in this case 90% of the solar radiation strikes the nucleus,

$$J_n = 0.9 J_e = 7.10 \times 10^{-3} \text{ cal cm}^{-2} \text{ sec}^{-1}, \quad (4)$$

while the rest is lost through absorption and scattering in the atmosphere.<sup>3</sup>

The Mies Theory must be used for scattering and absorption of dust in the cometary atmosphere. For dust particles with "radius"  $a$  and index of refraction close to one, for the average coefficient of efficiency  $q \approx 2$ , if  $x = 2\pi a/\lambda > 2$  (Van de Hulst, 1957). From this, for example, it follows that the extinction cross section (absorption and scattering) for dust with  $a$  assumed to be  $1.5 \times 10^{-5} \text{ cm}$  works out to

$$\sigma_s = \pi a^2 q \approx 1.4 \times 10^{-9} \text{ cm}^2, \text{ for } \lambda < \pi a = 4700 \text{ \AA}. \quad (5)$$

For longer wave lengths,  $\sigma_s$  rapidly drops to zero. For our estimates it is sufficient to let  $\sigma_s = 0$  for  $\lambda > \pi a$ .

An estimate for dust density can be obtained from the data for comet Arend-Roland (1957 III). Liller (1960) has found for the scattered light in the tail of this comet, a mass loss (with iron dust) of  $Q'_s \approx 8 \times 10^7 \text{ gr sec}^{-1}$ , a particle mass of  $m_s \approx 8 \times 10^{-13} \text{ gr}$  and a particle radius of  $a \approx 3 \times 10^{-5} \text{ cm}$ . If we assume that all of the dust particles coming from the nucleus enter the tail, the dust yield from the nucleus per spatial angle and unit time will be

$$Q_s = (1/4\pi) Q'_s / m_s \approx 8 \times 10^{18} \text{ sec}^{-1} \text{ sterad}^{-1}. \quad (6)$$

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<sup>3</sup>Diffuse reflection on the surface of the nucleus will be discussed later in detail.

From the two following considerations for the comet Arend-Roland we can see that this yield is of the correct order of magnitude.

1. Beyer (1958) gives approximately  $10'$  for the apparent coma diameter ( $2R_K/\Delta$ ) with  $\Delta = 1$  astronomical unit of geocentric distance, corresponding to a radius of  $2.2 \times 10^{10}$  cm. Assuming that the density in the coma changes as  $r^{-2}$ , we obtain for the number of dust particles in the area of the coma

$$N_s = \int_{R_K}^{R_K/\Delta} \frac{Q_s}{r^2} 4\pi r^2 dr \approx 2 \times 10^{23}, \quad (7)$$

for  $v = 10^5$  cm sec $^{-1}$  and the value of  $Q_s$  given in equation (6). Remy-Battiau (1964) calculates  $N_s = 3 \times 10^{23}$  to  $8 \times 10^{27}$  for various models. The value  $Q_s$  which is assumed therefore leads to essentially average  $N_s$  values.

2. The visual ray along which the coma can differ from the background, has a distance of  $R_K$  from the nucleus. The number of particles on this visual ray, if the density decreases as  $r^{-2}$ ,

$$N_s(R_K) \approx \pi \cdot Q_s / (R_K^2) \approx 10^3 \text{ cm}^{-2}. \quad (8)$$

For a visual ray which is perpendicular to the axis of the tail, we have

$$N_s(R_{\text{tail}}) \approx 2R_{\text{tail}} n_{\text{tail}}, \quad (9)$$

and if we assume that  $n_{\text{tail}} = 1.35 \times 10^{-8}$  cm $^{-3}$  according to Liller (1960), and further assume that the tail diameter is at least  $2R_{\text{tail}} \approx 3R_K$ ,  $N_s(R_{\text{tail}})$  will be approximately equal to  $10^3$  cm $^{-2}$ . The value assumed for  $Q_s$  therefore seems to be completely plausible.

For the assumed yield, the optical depth for  $\lambda < 4,700$  Å will then be (with  $a = 1.5 \times 10^{-5}$  cm)

$$\int_R^\infty \mu_s dr = \frac{\sigma_s Q_s}{v R} \approx 1.1 \times 10^3 / R = 0.11, \quad (10)$$

if we use a rough value of  $R = 10^6$  cm for the radius of the nucleus.

On the other hand, we find from the data of Houziaux (1959) that the intensity of the scattered light with isotropic scattering  $J_{\text{scatter}} \approx 4\pi \times 2.3 \times 10^{-11} (\Delta/R_K)^2$  erg cm $^{-2}$  sec $^{-1}$  Å $^{-1}$ . The intensity of the incident solar light is  $J_{\text{incident}} = 82$  erg cm $^{-2}$  sec $^{-1}$  Å $^{-1}$ , for  $\lambda \approx 4,700$  Å.

Since the number of scattering particles in a visual ray passing at a distance of  $s$  from the head, changes as  $s^{-1}$ , we will obtain for the scattered light in a cylinder which passes through the nucleus and has an area  $\pi R^2$ , where  $R$  is the radius of the nucleus,

$$\frac{J_{\text{scatter}}}{J_{\text{incident}}} = \frac{4\pi \times 2.3 \times 10^{-11} \times (4R_n)^2}{82} \frac{\left\{ \int_0^R [\pi Q_s(r/s)] 2\pi s ds \right\} (\pi R^2)}{(4\pi Q_s(R_n/r)) (\pi R_n^2)} \approx 5.7 \times 10^{-2}. \quad (11)$$

From this result we can now derive the optical depth: with an Albedo<sup>4</sup> of approximately 0.5, we will have for the ratio of extinction to incident radiation

$$\frac{J_{\text{extinction}}}{J_{\text{incident}}} = \frac{J_{\text{scatter}}/J_{\text{incident}}}{0.5} \approx 0.11 \quad (12)$$

and since

$$\frac{J_{\text{extinction}}}{J_{\text{incident}}} = 1 - e^{-\int_R^\infty \mu_s dr}, \quad (13)$$

the optical depth according to the observation data will be

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$$\int_R^\infty \mu_s dr \approx 0.12;$$

in good agreement with the optical depth calculated in equation (10).

Since the extinction cross section  $\sigma_s$  is proportional to  $a^2$  and the upper wave length limit for which equation (5) can be used, increases with  $a$ , the shielding is very sensitive to the size of the dust particles. For a value of  $a$  which is somewhat larger, we can therefore obtain a much larger optical depth. We must therefore take into account that the nucleus in the case of dusty comets can be shielded considerably by the sunlight.

The solar energy striking the nucleus of the comet heats not only the surface but also the interior, as was shown by Minnaert (1947), particularly

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<sup>4</sup>Ratio of the scatter to the extinction (Van de Hulst 1957, p. 183).

if the comet is still more than 2 or 3 astronomical units from the Sun. As it approaches the Sun, the volatile substances are evaporated. This evaporation counteracts the warming of the surface and keeps the temperature gradients small. It is therefore sensible to assume that the temperature of the layers near the surface is approximately 150° K (Donn and Urey, 1957) and that only very small amounts of heat reach the interior during the periods of evaporation. Table 1 lists the constants that can be used in the following calculations. If we wish to determine the change in heat of the interior, it is simply a question of making an appropriate change in the heat of evaporation.

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TABLE 1. HEAT VALUES AND TEMPERATURES

	C <sub>2</sub> H <sub>2</sub>	C <sub>2</sub> H <sub>4</sub>	C <sub>2</sub> H <sub>6</sub>	CH <sub>4</sub>	CO <sub>2</sub>	NH <sub>3</sub>	C <sub>2</sub> N <sub>2</sub>	H <sub>2</sub> O
Freezing point (°K)	192	104	90	90	217	198	239	273
Melting point (cal/mole)				233	1,980	1,840		1,440
Boiling Point T <sub>s</sub> at 1 ata (°K)	188	169	184	114	195	240	252	373
Heat of evaporation L at the boiling point (cal/mole)			3,800	2,200		5,550	5,300	9,700
Trouton constant L/(R <sub>0</sub> T <sub>s</sub> )			10.4	9.7		11.7	10	13.1
Heat of evaporation from the solid phase L' (cal/mole)	5,240	3,440		2,370	6,250	7,430	7,800	
C = 1'/(R <sub>0</sub> T <sub>s</sub> )	13.9	10.2		11.0	16.2	16.4	15.6	
Temperature range for L' and C (°K)	133-191	113-109		79-89	138-216	146-195	201-245	

Heat of desorption for most gases  $\approx 10^4$  cal/mole.  
 $R_0 = 1.99$  cal/(mole °K).

At the surface of the nucleus, the following energy is used for evaporation and emission per unit area and unit time:



$$(1 - A) J_v = \frac{Q}{R^2} \frac{L}{N_0} + (1 - A) \sigma T^4, \quad (14)$$

Here  $A$  is the diffuse reflection at the surface of the nucleus (Albedo),  $L$  is the heat of evaporation in cal/mole,  $N_0 = 6.025 \times 10^{23}$  molecules/mole, Loschmidt's number,  $\sigma = 1.35 \times 10^{-12}$  cal cm<sup>-2</sup> sec<sup>-1</sup> (°K)<sup>-4</sup> is the Stefan-Boltzmann constant. If we now assume energy equilibrium,

$$J_r \approx J_a / r_h^2, \quad (15)$$

with  $r_h$  equal to the heliocentric distance in astronomical units, we will have

$$\frac{Q}{R^2} = (1 - A) \left( \frac{J_a}{r_h^2} - \sigma T^4 \right) \frac{N_0}{L}. \quad (16)$$

The value for  $J_a$  is given in equation (4).

Since the outflowing gas is in temperature equilibrium with the surface, no additional energy will be consumed in heating the gas. The warming of the gas and the ice to equilibrium temperature constitute long term accumulations of energy which are used up to a large extent even at considerable distances from the Sun and therefore need not be taken into account in the energy balance. The liberated energy from chemical compounds is not taken into account; such energy, if it were liberated near the surface, would further increase gas production.

Using the Trouton rule, that the integration constant for the boiling point  $T_s$  is

$$L / (R_0 T_s) \approx 10 \quad (17)$$

( $R_0$  is the gas constant), we will obtain from the Clausius-Clapeyron equation for vapor pressure

$$P = 1.013 \times 10^6 \times e^{10 \cdot L / (R_0 T)} \text{ dyn cm}^{-2}. \quad (18)$$

For an ideal gas with  $N$  molecules per cm<sup>3</sup>,

$$N = P / (kT), \quad (19)$$

and the yield, with the aid of equation (19), is

$$\frac{Q}{R^2} = N v_T = P \sqrt{\frac{8 N_0}{\pi M k T}} \quad (20)$$

The average velocity of the outflowing gas is assumed to be  $v_T = [8 R_0 T / (\pi M)]^{1/2}$  cm sec<sup>-1</sup>. By substituting equation (18) into equation (20), with a molecular weight  $M = 18$ , we will obtain

$$\frac{Q}{R^2} = \frac{1}{T} \times 10^{29.744 - 0.2182 L/T} \quad (21)$$

For low temperatures where one phase is ice, according to Table 1 we will have

$$L' / (R_0 T_s) \approx 15 \quad (22)$$

and then obtain

$$\frac{Q}{R^2} = \frac{1}{T} \times 10^{31.915 - 0.2182 L/T} \quad (23)$$

The task now is to calculate  $Q/R^2$  as the function of  $T$  and  $L$  (or  $L'$ ) from equations (16) and (21) (or 23) which must be simultaneously satisfied. The solution to the transcendent equation thus obtained is shown in Figure 1, where the Albedo  $A$  is equated to 0 and equation (16) is plotted only for  $r_h = 1$  astronomical unit. As we can see, for  $r_h = 1$  astronomical unit,  $Q/R^2 > 10^{17}$  molecules cm<sup>-2</sup> sec<sup>-1</sup> sterad<sup>-1</sup>, and for highly volatile substances even  $> 10^{18}$ . With a nuclear radius of  $R = 10^6$  cm,  $Q$  will even be greater than  $10^{29}$  molecules sec<sup>-1</sup> sterad<sup>-1</sup>, or  $> 10^{30}$ , in agreement with Biermann and Trefftz (1964). Here equation (21) is employed. If equation (23) were used instead, the end result would not be considerably changed; the steep slope of the curves in Figure 2 (Figure 2 is obtained directly from the resultant curves of Figure 1), would then occur only at somewhat larger values of  $L$  (with larger  $r_h$ ). In both figures the heliocentric distances have been selected so that the  $r_h$  which is of interest is multiplied times  $\sqrt{2}$  and we can then read off  $Q/R^2$ , assuming that there is a shielding of 50% of the solar radiation by the dust in question.



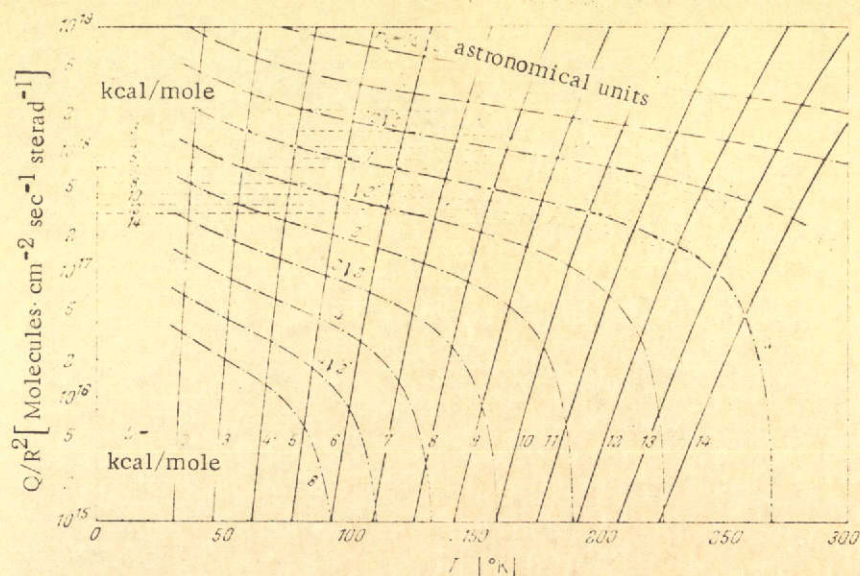


Figure 1. Molecular Yield  $Q/R^2$  per  $\text{cm}^2$  of Nucleus Surface, Spatial Angle 1 and sec as a Function of Heliocentric Distance  $r_h$  in Astronomical Units Above Evaporation Temperature  $T$ .

$$\text{-----} \left( \frac{a}{r_h^2} - \sigma T^4 \right) \frac{N_0}{L}, \quad r_h = 1 \text{ astronomical unit};$$

$$\text{-----} \frac{1}{\sqrt{T}} \cdot 10^{29.744 - 0.2182 L/T}; \quad \text{-----} R/Q^2 \text{ for constant values of } r_h.$$

### III. COMPARISON WITH OBSERVATIONS

$Q/R^2$ , but as a function of  $r_h$  with  $L$  as a parameter, is shown in Figure 2. The brightness of comets can be compared with these curves if we assume that the brightness is proportional to the total number of molecules in the cometary atmosphere and the dilution factor  $r_h^{-2}$ . The brightness is then a function of density and coma size. The size of the visible coma however depends in turn upon the density. If the density drops off constantly with  $r^{-2}$  from the nucleus, and a molecular density at the surface of the nucleus

$$n_0 = Q/(vR^2) \quad (24)$$

increases, the radii of all the isophots will increase and therefore the radius of the visible coma will increase as well; the number  $N [\text{cm}^{-2}]$  of the



particles in a visual ray, which passes through a given isophot, must remain /30  
constant. The brightness of the coma up to a radius  $s$  is (with  $s \gg R$ )

$$J = (K/r_h^2) \int_R^s n_0 (R/r)^2 4\pi r^2 dr = K' n_0 s^2 / r_h^2, \quad (25)$$

where  $K$  and  $K'$  are proportionality constants. The radius of a given isophot is obtained from the appropriate value of  $N$ , similar to the situation in equation (8) and with the aid of equation (24), i.e.,

$$s = \frac{\pi Q}{c A^2} = \frac{\pi R^2}{A^2} n_0. \quad (26)$$

By substituting equation (26) in equation (25) we will have

$$J = K'' n_0^2 / r_h^2, \quad (K'' = K\pi R^2 / A^2). \quad (27)$$

If the density increases as  $r^{-2}$  up to a radius  $s = R_K$ , at which the coma can just be made out against the background (this is also usually the radius at which the observer can delimit the coma by using a diaphragm), equation (27) will be valid for the brightness of the coma. The comet 1941 I, according to data of Vsekhsvyatskii (1964), is similar for different solar distances, as required by equation (27). The distribution of the luminous molecules, however, generally decreases more rapidly toward the "edge" of the coma than  $r^{-2}$  (Haser, 1957), and the exponent of  $n_0$  in equation (27) will therefore be between 1 and 2.

With the usual disregard of the phase function, the brightness will usually be written as

$$J = \frac{J_0}{\Delta^2 r_h^2} \quad (28)$$

or in size classes,

$$m = m_0 + 5 \log \Delta + 2.5 \log r_h. \quad (29)$$

From equation (27) we obtain

$$m = m'_0 - 5 \log n_0 + 5 \log r_h. \quad (30)$$



We can now refer to two cases:

1.  $J_a/r_h^2 \gtrsim \sigma T^4$ ; equation (16) then becomes

$$\frac{J_a}{r_h^2} \gg \frac{1}{(1-A)} \frac{Q}{R^2} \frac{L}{N_0}.$$

From the generalization of equations (21) and (23)

$$\frac{Q}{R^2} = \frac{1}{V^2 T} \times 10^{\alpha - \beta L/T}, \quad (31)$$

we then obtain, using equation (24)

$$\log n_0 \approx \gamma - \beta L (\sigma/J_a)^{1/4} r_h^{1/2}, \quad (32)$$

and after substitution, equation (30) becomes

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$$m \approx A + 5 \beta L (\sigma/J_a)^{1/4} r_h^{1/2} + 5 \log r_h. \quad (33)$$

From equation (29) we obtain

$$v = \frac{1}{2.5} \frac{dm}{d(\log r_h)} = \ln 10 \times \beta L (\sigma/J_a)^{1/4} r_h^{1/2} + 2 \approx 1.87 \times 10^{-3} L r_h^{1/2} + 2, \quad (34)$$

or, in comparison with the Levin formula<sup>5</sup>,

$$v = L r_h^{1/2} / (R_0 T_0) + 2, \quad (35)$$

where  $T_0 = (J_a/\sigma)^{1/4}$  and  $\beta = 1/(R_0 \ln 10)$ . In equation (35),  $L$  is therefore made smaller by a factor of 2 than in the Levin formula in order to get the same value for  $v$  which is reduced by the cometary data. Equations (34) and (35) are only valid when  $J_a/r_h^2 \gtrsim \sigma T^4$ ; therefore, if  $r_h$  or  $L$  is large, as can be seen from an examination of Figure 1.

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<sup>5</sup>See for example Richter (1954, 1963):  $v = 1/2 L r_h^{1/2} / (R_0 T_0)$ . The term +2, which comes from the dilution factor, must be added separately.



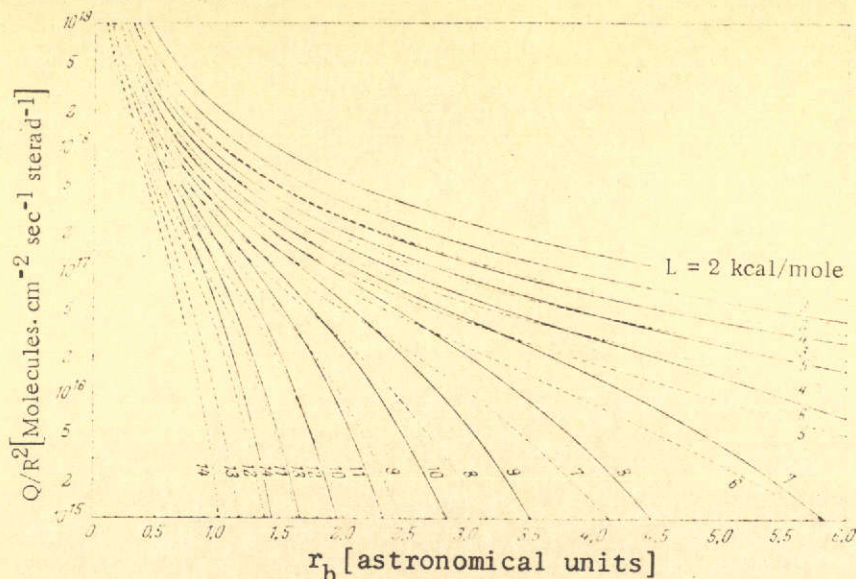


Figure 2. Molecular Yield  $Q/R^2$  per  $\text{cm}^2$  of Nucleus Surface, Spatial Angle  $l$  and  $\text{sec}$  as a Function of Heat of Evaporation  $L$  Over Heliocentric Distance  $r_h$  in Astronomical Units.

———— Solar energy incident on the nucleus =  $J_a/r_h^2$ ;  
 ----- Incident solar energy weakened by the dust in the atmosphere, by 50%.

2.  $J_a/r_h^2 \gg \sigma T^4$ , equation (16) is then approximately

$$\frac{Q}{R^2} \approx (1 - \epsilon) \frac{J_a}{r_h^2} \frac{N_0}{L}, \quad (36)$$

and since according to equation (24) and (27)  $J$  is proportional to  $(Q/R^2)^2/r_h^2$ , we will have

$$v = 6. \quad (37)$$

For density distributions in a coma which drop off more rapidly toward the "edge" than  $r^{-2}$ ,  $v$  must be between 4 and 6. According to the data of Vsekhsvyatskii (1964)<sup>6</sup>  $v > 4$  comets with perihelial distance  $< 1$  astronomical unit.

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<sup>6</sup>See for example page 35 [of the German text].

For  $v = 4$ , we must reduce the  $J^{1/2}$  values (for  $v = 6$ :  $J^{1/3} =$  values) from observations to  $\Delta = 1$  astronomical unit and compare with the curves in Figure 2. Comets which have been observed for a long time so that their  $r_h$  changes over several astronomical units, then provide information about  $Q/R^2$  and also about  $L$ ; however, even for slight changes in  $r_h$  we can get quite good estimates for  $Q/R^2$ . Particular interest attaches to those comets which are being discovered for the first time at large  $r_h$ ; for example Comet Humason 1962 (1961 e) or Comet Bester (1948 I), which had  $L \approx 3.5$  kcal/mole prior to passage through perihelion and  $L \approx 5.5$  kcal/mole after passage through perihelion. In Table 2, we have summarized 20 observations of comets with estimates for  $L$  (before perihelion)/ $L$  (after perihelion) and  $Q/R^2$ . Most comets in which the forbidden [OI]-lines have been observed, are included. Figure 3 shows that perihelion values for  $J^{1/2}$  over  $Q/R^2$  for the 20 comets. For  $J^{1/3}$ ,  $Q/R^2$  is approximately the same but  $L$  is somewhat larger. The letter  $s$  is the  $L$ -column of Table 2 and in Figure 3 indicates that in these comets 50% shielding of solar radiations by dust was assumed.

If we compared the observations of the forbidden [OI]-lines with  $J^{1/2}$  or  $Q/R^2$ , as indicated in Figure 3, we will find in general that the red lines are strong and the green lines weak (or decreasing), if  $Q/R^2 > 10^{18}$  ( $Q > 10^{30}$  for  $R = 10^6$ ). On the other hand, the green line becomes strong and the red doublet weak when  $10^{18} > Q/R^2 > 10^{17}$ . The data for the strength of the lines come from Swings (1962) and Remy-Battiau (1962). It should be pointed out that in the case of Comet 1941 IV, 1961 e, 1943 I and 1952 I, although they fall in the [OI]-line region of Figure 3, no such lines were observed. The appearance of a strong green line at 5577 Å, while red doublet 6300 Å and 6364 Å is weak, cannot be explained by a depopulation of the  $^1D_2$  level; at still higher densities, when depopulation would be even more pronounced due to impacts, the intensity ratio is exactly reversed.

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TABLE 2. COMPARISON OF COMETS

Name (No. of Observation)	Designation	$J^{1/2*}$	$Q/R^2$ $\text{cm}^{-2} \text{sec}^{-1}$ $\text{sterad}^{-1}$	$L_v/L_n^{**}$ kcal	Notes
1 Finsler	1937 V	$1.1 \times 10^{-1}$	$1 \times 10^{18}$	6/	Long observation before perihelion
2 Cunningham	1941 I	$1.5 \times 10^{-1}$	$6 \times 10^{18}$	4/	
3 de Kock	1941 IV	$9.6 \times 10^{-2}$	$1.5 \times 10^{18}$	/7	Long observation after perihelion
4 van Gent	1941 VIII	$5.0 \times 10^{-2}$	$7 \times 10^{17}$	6/8	
5 Whipple-Fedtke	1943 I	$8.4 \times 10^{-2}$	$2 \times 10^{18}$	14/10	Comet divided after passage through perihelion
6 Encke (42)	1947 XI	$7.6 \times 10^{-2}$	$9 \times 10^{17}$	/10.5 <sub>s</sub>	
7 Bright Southern Comet	1947 XII	$5.7 \times 10^{-1}$	$5 \times 10^{18}$	/9	Long observation before and after perihelion
8 Bester	1948 I	$9.0 \times 10^{-2}$	$1.5 \times 10^{18}$	3.5/5.5	
9 Honda	1948 IV	$1.0 \times 10^{-1}$	$9 \times 10^{17}$	/11	Long observation after perihelion
10 Large Southern Comet of the ecliptic	1948 XI	$3.5 \times 10^{-1}$	$2 \times 10^{18}$	/9	
11 Johnson	1950 I	$9.1 \times 10^{-3}$	$3 \times 10^{17}$	5/5	Perihelial distance 2.56 astro. units.
12 Minkowski	1951 I	$2.9 \times 10^{-2}$	$2 \times 10^{17}$	5/6	
13 Encke (43)	1951 III	$3.3 \times 10^{-2}$	$8 \times 10^{17}$	8 <sub>s</sub> /8 <sub>s</sub>	Long observation before perihelion
14 Wilson	1952 I	$4.7 \times 10^{-2}$	$2 \times 10^{18}$	6/6	
15 Schaumasse (5)	1952 III	$1.8 \times 10^{-2}$	$1 \times 10^{17}$	10 <sub>s</sub> /10 <sub>s</sub>	Long observation before and after perihelion
16 Comet Sola (4)	1952 VII	$4.5 \times 10^{-3}$	$8 \times 10^{16}$	6 <sub>s</sub> /6 <sub>s</sub>	
17 Arend-Roland	1957 III	$4.9 \times 10^{-1}$	$7 \times 10^{18}$	5/5	Long observation before perihelion
18 Mrkos	1957 V	$5.0 \times 10^{-1}$	$4 \times 10^{18}$	/3	
19 Encke (44)	1957 VIII	$3.5 \times 10^{-2}$	$8 \times 10^{17}$	/10 <sub>s</sub>	Long observation before perihelion
20 Humason	1961 e	$8.7 \times 10^{-2}$	$5 \times 10^{17}$	3/	

\*  $J = \Delta^2 10^{-0.4 m}$ ; with  $\Delta$  = geocentric distance,  $m$  = size class at perihelion.

\*\*  $L_v = L$  before perihelion,  $L_n \dots L$  after perihelion.



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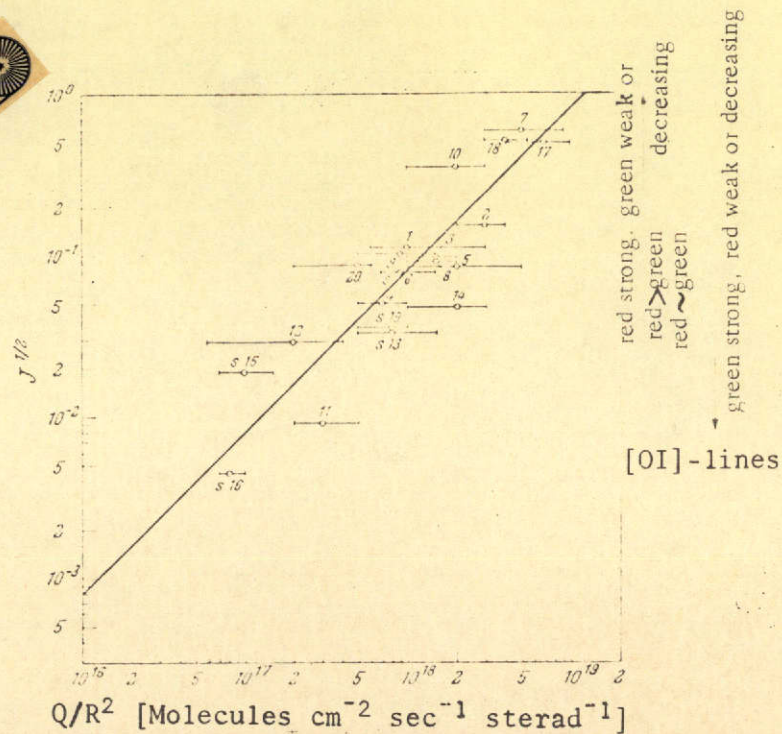


Figure 3. Measurement of Brightness  $J^{1/2} = \Delta \cdot 10^{-0.2 m}$  Over  $Q/R^2$  for the 20 Comets Listed in Table 2. The Forbidden [OI]-Lines are Indicated on the Right Side.



Figure 4. Term Diagram of the Forbidden [OI]-Lines.

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